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Overhead catenary system-pantograph coupled system dynamics modelling and analysis

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Abstract. Based on the static and dynamic characteristic of catenary system, vibration model of pantograph and catenary coupling system is presented, and simulation calculation is performed. In every time step, the traversal algorithm is used to solve dynamic equations of catenary system model and linear pantograph model, and the estimation of the contact condition of pantograph and catenary coupling system is performed, and enter the next time step. On the occasion of double pantograph being used, the dynamic equations are solved in turn. According the results of simulation calculation, the relationships between design parameters of pantograph and catenary system and the quality of current collection are analyzed.

Key words. High speed railway, overhead catenary system, pantograph-catenary system, dynamics, modelling, simulation.

1. Introduction

Generally, the mathematical simulation model of the Pantograph-OCS (Overhead Catenary System) system could be used to simulate the fluctuation of the contact force of the pantograph, calculate the parameter that is used for evaluating the quality of quality of the current collection of the Pantograph-OCS system, estimate the highest running speed of the locomotive, appraise the wear phenomenon of the Pantograph-OCS system and analyze and design the active control pantograph. The literature [1] points out that it is available to simulate required characteristics of the simulation system model of the Pantograph-OCS system. The literature [2]

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indicates that the linear dynamical model of the time-varying stiffness coefficient of a single-degree-of-freedom belt has been used to simulate the catenary system. On that basis, literature [3] shows that the contact model of the Pantograph-OCS system has been simplified and it is more practical to simplify the Pantograph-OCS system as a beam structure with moving force load and axial stress rather than a cable structure with moving load. Literature [4] an analytical form equation has been established for the beam model with moving force load and axial stress and the solution of the equation had been found out. As a result, to improve the design of the high-speed electric railway, literature [5] suggests that a simulation model is supposed to be established for the Pantograph-OCS system by the mixed method in tension sections. Literature [6] indicates that the flexible contact line has been modelled as the nonlinear continuous beam structure together with absolute coordinate. Literature [7] the simulation model of the catenary system has been established by the finite element computing method and by virtue of Euler-Bernoulli beam theory. Literature [8] reveals that the analysis of the Semi-analytical model has been carried out, whose results have been compared to that produced by the finite element computation model. In addition, the model raised in literature [9] has been improved. In order to get accurate dynamic response, an appropriate initial equilibrium state is a must for the dynamic simulation model. The most commonly used method to calculate the static stiffness distribution of the catenary system is the finite element computing method. [10, 11] However, just as it is showed in literature [12], it is a highly complex process to carry out this kind of calculation upon the catenary system by the finite element computing method. Hence, a rapid method with high accuracy and robustness is needed to establish the static stiffness distribution of the catenary system.

2. Overhead catenary system-pantograph coupled system dynamics model

2.1. Dynamics model of pantograph system

Pantograph system includes body frame, upper arm, lower arm, pantograph bow, and gearing. During the interaction of overhead catenary system–pantograph coupled system, the upper arm and lower arm of pantograph system indicate translational motion and rotary movement. In order to simply the problem, and reflect the main dynamic traits in the overhead catenary system–pantograph coupled system, average research tends to conduct linear processing in the pantograph system. The main idea is to ignore vibration of locomotive, and simplify translational and rotary movements of upper and lower arms in pantograph system to vertical movements within a height range. Moreover, quadratic terms are overlooked in calculation, so as to obtain the linearized model with a height range. Figure 1 is a type of linearized model of pantograph system, which simplifies the pantograph system to quality-spring-damping model.

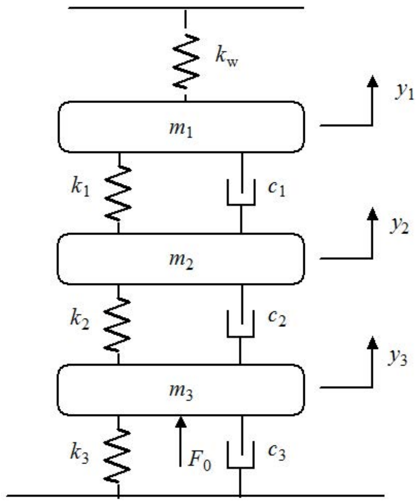


Fig. 1. Linearized model of pantograph system

The equilibrium equation of pantograph system is:

$$m_1\ddot{y}_1 + c_1\dot{y}_1 - c_1\dot{y}_2 + (k_1 + k_w)y_1 - k_1y_2 = 0\,, \tag{1}$$
$$m_2\ddot{y}_2 - c_1\dot{y}_1 + (c_1 + c_2)\dot{y}_2 - c_2\dot{y}_3 - k_1y_1 + (k_1 + k_2)y_2 - k_2y_3 = 0\,, \tag{2}$$
$$m_3\ddot{y}_3 - c_2\dot{y}_2 + (c_2 + c_3)\dot{y}_3 - k_2y_2 + (k_2 + k_3)y_3 = F_0\,. \tag{3}$$

m_i, k_i, c_i $(i = 1, 2, 3)$ are pantograph bow, equivalent mass of upper frame and lower frame, equivalent stiffness and equivalent damping. K_w is the stiffness of overhead catenary system at the position of pantograph. F_0 is the static lifting force. $F(t)$ is the dynamic contact force of bow net. Equations (1) – (3) can be converted to:

$$\mathbf{M}\ddot{\mathbf{Y}}(t) + \mathbf{C}\dot{\mathbf{Y}}(t) + \mathbf{K}\mathbf{Y}(t) = \mathbf{F}_e(t)\,. \tag{4}$$

$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$ is mass matrix, $\mathbf{C} = \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & c_1 + c_2 & -c_2 \\ 0 & -c_2 & c_2 + c_3 \end{bmatrix}$ is damping matrix, $\mathbf{K} = \begin{bmatrix} k_1 + k_w & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ 0 & -k_2 & k_2 + k_3 \end{bmatrix}$ is stiffness matrix, $\mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$, $\dot{\mathbf{Y}} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix}$, $\ddot{\mathbf{Y}} = \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{bmatrix}$ are displacement, velocity and acceleration vector $\mathbf{F}_e = \begin{bmatrix} 0 \\ 0 \\ F_0 \end{bmatrix}$ is lifting victor.

2.2. Dynamics model of OCS system

The analysis shown below is based on the kinetic model for OCS under operation conditions of the single-arm pantograph. In order to simplify the problem, a constant lifting force is adopted in the model to replace the effect of the pantograph system on the OCS.

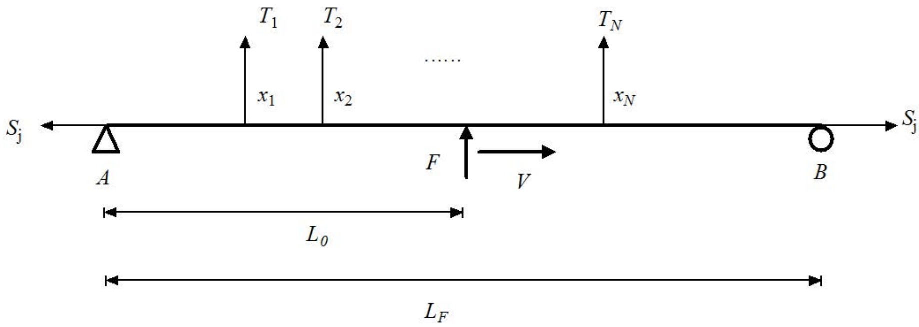


Fig. 2. The model for pantograph coupling system under operation conditions of the single-arm pantograph

The relations between the pantograph and Overhead Contact Line (OCL) at a certain point of time is shown in Figure 2. The Endpoint A and B represent two terminals of the OCL vibration distortion. In fact, there are no points of support existing at A or B on the OCL, furthermore, vibration distortion does not exist at other parts apart from these two points on OCL. Thus, these two points are assumed as points of support, and the displacement restraint can be realized in the model. $T_1, T_2, \ldots, T_N$ are the dropper tension distributed between A and B on the OCL, and $x_1, x_2, \ldots, x_N$ represent the positions of the dropper tension that are relative to the pantograph, and N represents the amount of droppers between A and B. F represents the lifting force of the pantograph, that moves along the OCL with a horizontal velocity of V. S_j is the horizontal tension of OCL. L_F represents the OCL length of the vibration tension, L_0 represents the pantograph position between A and B on the OCL that could lead to vibration distortion. In order to calculate the vibration of the pantograph system at a certain point of time as shown in Figure1, let's assume the movement distance of pantograph with a velocity of V starting from A along the OCL $L_0 = Vt$. According to the wave equation of the catenary model, we can have

$$
\begin{aligned}
m_j \frac{\partial^2 y(x,t)}{\partial t^2} - S_j \frac{\partial^2 y(x,t)}{\partial x^2} = \frac{F}{l_F} \cdot (u(x-(Vt-\frac{l_F}{2})) - u(x-(Vt+\frac{l_F}{2}))) \\
+ \sum_{i=1}^{N} \frac{T_i}{l_T} \cdot (u(x-(Vt+x_i-\frac{l_T}{2})) \\
- u(x-(Vt+x_i+\frac{l_T}{2}))) \,.
\end{aligned} \tag{5}
$$

According to the solution calculation process, we can have

$$y(x,t)=\sum_{k=1}^{\infty}(A_k\sin\frac{k\pi l_F}{2L_F}(\sin\frac{k\pi Vt}{L_F}-\frac{V}{\sqrt{S_j/m_j}}\sin(\sqrt{\frac{S_j}{m_j}}\frac{k\pi t}{L_F})))\sin\frac{k\pi x}{L_F}\,, \tag{6}$$

$$A_k=\frac{4(F\pm F_r)L_F^2/m_j}{(k\pi)^3 l_F(S_j/m_j-V^2)}\,, \tag{7}$$

we can have the counterforce of the dropper reflected wave when the front pantograph moves to a certain point of the OCL under the double-arm pantograph condition F_{rq},

$$F_{rq}=c_q\cdot F_q+c_h\cdot F_h=r\cdot(c_q/a^2+c_h\cdot a^2)\cdot F+r\cdot(c_q/a^2+c_h\cdot a^2/|r|)\cdot F'\,, \tag{8}$$

we can have the counterforce of the dropper reflected wave when the back pantograph moves to a certain point of the OCL under the double-arm pantograph condition F_{rh},

$$F_{rh}=c_q\cdot F_q+c_h\cdot F_h=r\cdot(c_q/a^2+c_h\cdot a^2)\cdot F+r\cdot(c_q/a^2/|r|+c_h\cdot a^2)\cdot F'\,. \tag{9}$$

In order to calculate F_{rq} and F_{rh}, F' must be determined. The model does not take the damping characteristics of OCS into consideration, therefore for the OCL vibration displacement caused by lifting forces from the front and back pantographs, the amplitudes are the same but phases differ, thus, $0 < F' < F$. In order to compare influences received by the front and back pantograph respectively under the operation condition of double-arm pantograph, the difference regarding the absolute value of the dropper reflected wave counterforce when the front and back pantographs move to a certain point on the OCL is calculated as follows

$$F_{rh}-F_{rq}=(c_q/a^2-c_h\cdot a^2)(1-|r|)\cdot F'\,. \tag{10}$$

As the reflection coefficient $|r|<1$, thus the counterforce received by the back pantograph is higher than the front pantograph, and the counterforce differences received by the double-arm pantograph become larger when approaching the front dropper. Additionally, as the reflection coefficient $|r|$ decreases, the counterforce differences of the dropper reflected wave received by the front and back pantographs rises. After calculating the counterforce received by the pantograph from the dropper reflected wave, the vibration displacement of the OCS under the double-arm pantograph condition can be directly calculated, however, in terms of the front pantograph

$$A_k=\frac{4(F\pm F_{rq})L_F^2/m_j}{(k\pi)^3 l_F(S_j/m_j-V^2)}\,. \tag{11}$$

Regarding the back pantograph

$$A_k=\frac{4(F\pm F_{rh})L_F^2/m_j}{(k\pi)^3 l_F(S_j/m_j-V^2)}\,. \tag{12}$$

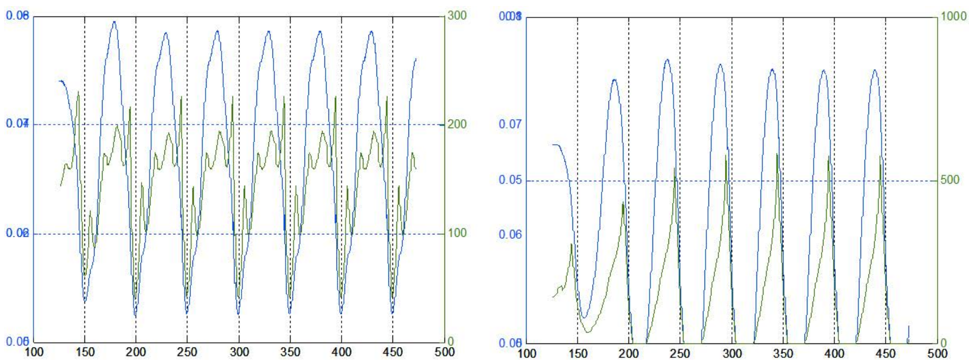
3. Overhead catenary system-pantograph coupled system dynamics simulation model

3.1. Overhead catenary system-pantograph coupled system dynamics model at single pantograph

The expression of contact line vibration displacement is given (6). As is shown in Figure 1, to calculate the kinematic equation of overhead catenary system–pantograph coupled system, it actually needs to solve the kinematic equation set by equations (6) and (4). There are many ways to use the numerical computation method in solving the kinematic equation of pantograph system, such as Newmark method, Wilson-θ method and central difference method. The central difference method makes the following assumptions on the velocity and accelerated speed at t_k.

$$[\frac{\mathbf{M}}{(\Delta t)^2} + \frac{\mathbf{C}}{2\Delta t}]\mathbf{Y}_{k+1} = \mathbf{F}_{ek} - [\mathbf{K} - \frac{2\mathbf{M}}{(\Delta t)^2}]\mathbf{Y}_k - [\frac{\mathbf{M}}{(\Delta t)^2} - \frac{\mathbf{C}}{2\Delta t}]\mathbf{Y}_{k-1}\,. \tag{13}$$

According to Equation (13), it is feasible to obtain the displacement $\mathbf{Y}_{k+1}$ at t_{k+1}. The simulation results are shown in Figure 3 and Figure 4. In these Figures, the horizontal axis is the pantograph horizontal displacement (m), the vertical axis on the left is the pantograph vertical displacement (m), and the vertical axis on the right is catenary-pantograph contact force (N).



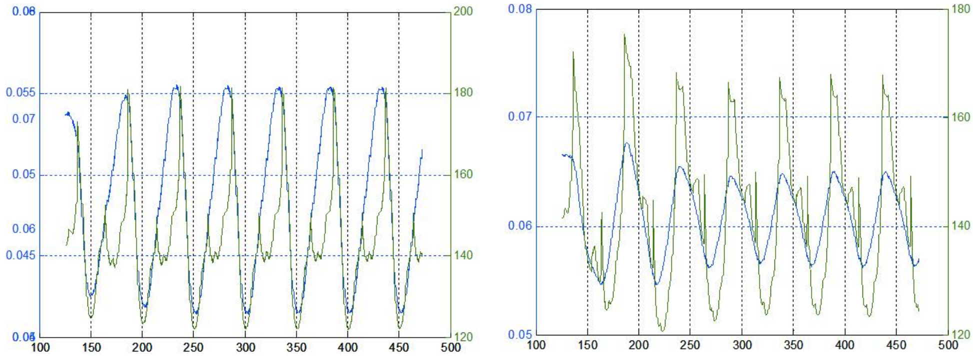
a) Simulation results at 250 km/h   b) Simulation results at 350 km/h

Fig. 3. Simulation results of catenary-pantograph coupling system vibration of simply link

3.2. Overhead catenary system-pantograph coupled system dynamics model at couple pantograph

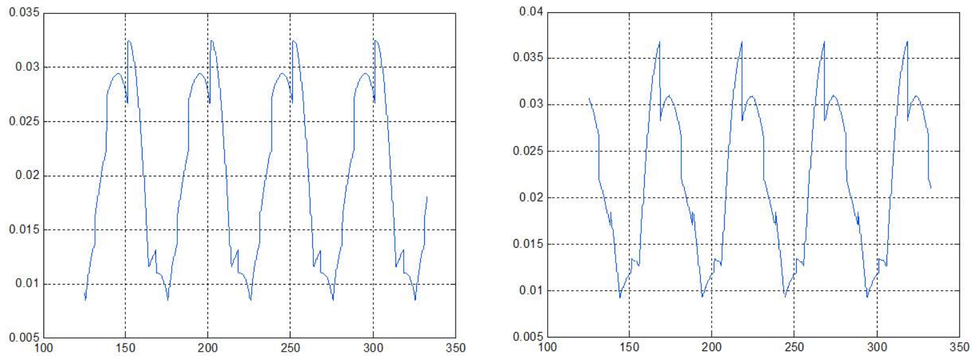
Equations (6), (8), (9), (11) and (12) are used to build overhead catenary system–pantograph coupled system dynamics model with lift force at couple pantograph. The simulation results are shown in Figure 5 and Figure 6.

It is more complicated to solve the kinematic equation of overhead catenary



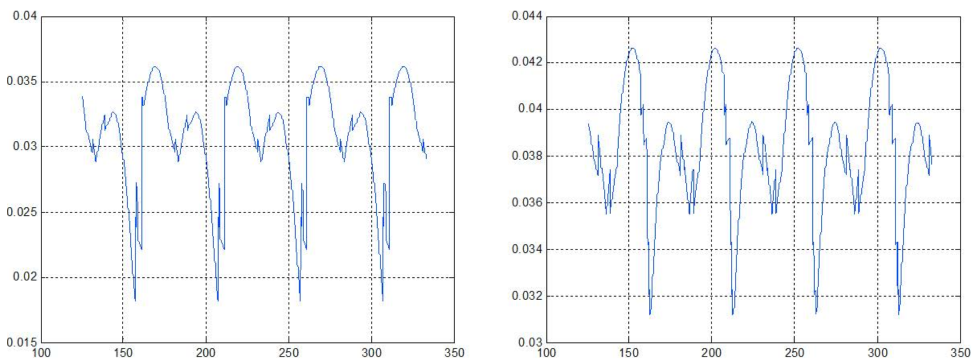
a) Simulation results at 250 km/h b) Simulation results at 350 km/h

Fig. 4. Simulation results of catenary-pantograph coupling system vibration of ammunition link



a) Contact line vibration at front pantograph b) Contact line vibration at back pantograph

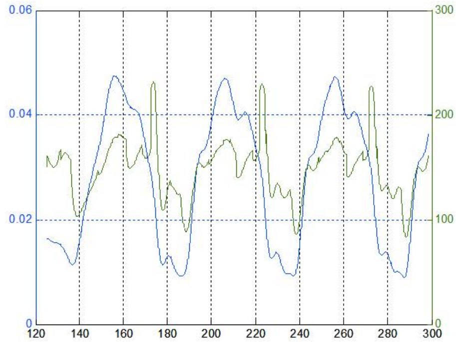
Fig. 5. Vibration of simple link 250 km/h at lift force of 116 N



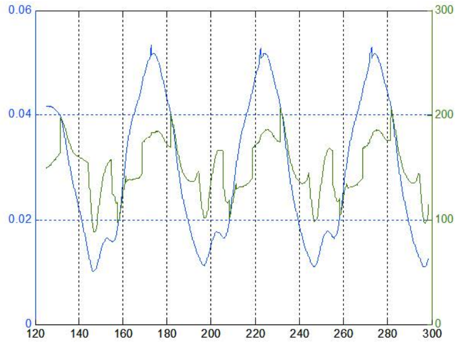
a) Contact line vibration at front pantograph b) Contact line vibration at back pantograph

Fig. 6. Vibration of ammunition link 250 km/h at lift force of 116 N

system–pantograph coupled system dynamics model at couple pantograph than single pantograph. In order to simplify the problem, fixate the state of motion for one pantograph before solving another kinematic equation of pantograph. As long as the time interval between the two adjacent solutions, the results can meet the requirements. The simulation results are shown in Figure 7 and Figure 8.

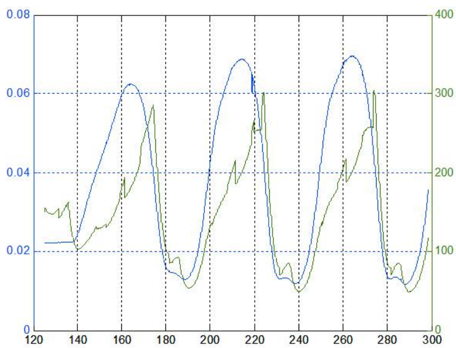


a) Lifting and contact force at anterior pantograph

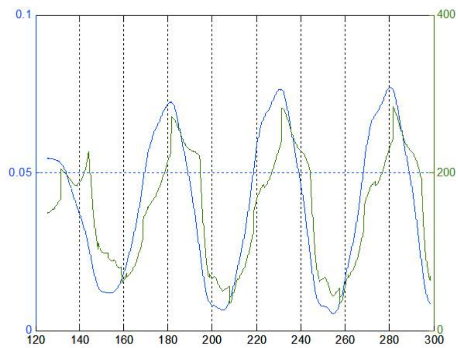


b) Lifting and contact force at posterior pantograph

Fig. 7. Simulation results of pantograph-catenary coupling system vibration at 250 km/h of simple link



a) Lifting and contact force at anterior pantograph



b) Lifting and contact force at posterior pantograph

Fig. 8. Simulation results of pantograph-catenary coupling system vibration at 350 km/h of simple link

4. Conclusion

Linearized pantograph model is combined with the dynamic vibration model of overhead catenary system to deduce the simulation algorithm of catenary-pantograph coupling vibration. Simulating calculation is conducted respectively when simple-chain suspension system and elastic suspension system operate at single and double pantograph. On this basis, the impacts of pantograph design parameters and overhead catenary system design parameters on current-receiving quality of pantograph are also analyzed. The dynamic property model of the OCS is proposed by combining the catenary structural force model and the mode superposition theory, which can effectively manifest the influences of various design parameters of the OCS on the dynamic property of the OCS.

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